Technical Report 1: On Gradient Descent Optimization in Diffusion-Advection based 3-D Molecular Cooperative Communication

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I. Convex Optimization Problem to Optimize the Number of Molecules

The lower bound of end-to-end probability of error $\overline{P}_e[j+1]$ considering Genie-aided approach [1] with EGC at the DN in $(j+1)$th time-slot is given by

$$\overline{P}_e[j+1] \geq \sum_{\mathbf{X}_1^{j-1} \in \chi} \Pr(\mathbf{X}_1^{j-1}) P_e[j+1|\mathbf{X}_1^{j-1}], \tag{1}$$

where the conditional probability of error $P_e[j+1|\mathbf{X}_1^{j-1}]$ at DN in $(j+1)$th time-slot for a given realization of $\mathbf{X}_1^{j-1}$ can be derived as

$$P_e[j+1|\mathbf{X}_1^{j-1}] \geq \beta_1 P_e[j+1|x[j]=1, \mathbf{X}_1^{j-1}] + \beta_0 P_e[j+1|x[j]=0, \mathbf{X}_1^{j-1}], \tag{2}$$

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where the terms $P_e[j+1|x[j]{=}1,\mathbf{X}_1^{j-1}]$ and $P_e[j+1|x[j]{=}0,\mathbf{X}_1^{j-1}]$ are given as

$$\begin{aligned}
&P_e[j+1|x[j]=1,\mathbf{X}_1^{j-1}]=\\
&\quad \Pr(N_{sc}[j]<\eta_c[j]|x[j]=1,\mathbf{X}_1^{j-1})\Pr(N_{egc}[j+1]<\eta_d[j+1]|x[j]=1,\widehat{x}[j]=0,\widehat{\mathbf{X}}_1^{j-1})\\
&\quad +\Pr(N_{sc}[j]\geq\eta_c[j]|x[j]=1,\mathbf{X}_1^{j-1})\Pr(N_{egc}[j+1]<\eta_d[j+1]|x[j]=1,\widehat{x}[j]=1,\widehat{\mathbf{X}}_1^{j-1})\\
&\quad =(1-P_{sc}^{D}[j|\mathbf{X}_1^{j-1}])P_{egc}^{10}[j+1|\widehat{\mathbf{X}}_1^{j-1}]+P_{sc}^{D}[j|\mathbf{X}_1^{j-1}]P_{egc}^{11}[j+1|\widehat{\mathbf{X}}_1^{j-1}],
\end{aligned}\tag{3}$$

$$\begin{aligned}
&P_e[j+1|x[j]=0,\mathbf{X}_1^{j-1}]=\\
&\quad \Pr(N_{sc}[j]\geq\eta_c[j]|x[j]=0,\mathbf{X}_1^{j-1})\Pr(N_{egc}[j+1]\geq\eta_d[j+1]|x[j]=0,\widehat{x}[j]=1,\widehat{\mathbf{X}}_1^{j-1})\\
&\quad +\Pr(N_{sc}[j]<\eta_c[j]|x[j]=0,\mathbf{X}_1^{j-1})\Pr(N_{egc}[j+1]\geq\eta_d[j+1]|x[j]=0,\widehat{x}[j]=0,\widehat{\mathbf{X}}_1^{j-1})\\
&=P_{sc}^{F}[j|\mathbf{X}_1^{j-1}](1-P_{egc}^{01}[j+1|\widehat{\mathbf{X}}_1^{j-1}])+(1-P_{sc}^{F}[j|\mathbf{X}_1^{j-1}])(1-P_{egc}^{00}[j+1|\widehat{\mathbf{X}}_1^{j-1}]),
\end{aligned}\tag{4}$$

respectively. Now using (3) and (4), the closed-form expression for $P_e[j+1|\mathbf{X}_1^{j-1}]$ in terms of Gaussian CDF is obtained as

$$P_e[j+1|\mathbf{X}_1^{j-1}]=\frac{\beta_1}{4}(A+B)+\frac{\beta_0}{4}(C+D),\tag{5}$$

where $A{=}[1+\Xi(\eta_c[j],\lambda_{sc}^{1}[j])][1+\Xi(\eta_d[j+1],\lambda_{egc}^{10}[j+1])]$, $B=[1-\Xi(\eta_c[j],\lambda_{sc}^{1}[j])]+[1+\Xi(\eta_d[j+1],\lambda_{egc}^{11}[j+1])]$, $C=[1+\Xi(\eta_c[j],\lambda_{sc}^{0}[j])][1-\Xi(\eta_d[j+1],\lambda_{egc}^{00}[j+1])]$, $D=[1-\Xi(\eta_c[j],\lambda_{sc}^{0}[j])][1-\Xi(\eta_d[j+1],\lambda_{egc}^{01}[j+1])]$ and the function $\Xi(n,\lambda)$ is defined as $\Xi(n,\lambda)=\mathrm{erf}\big(\frac{n-0.5-\lambda}{\sqrt{2\lambda}}\big)$.

Theorem 1. *The constrained convex optimization problem to obtain the jointly optimal number of transmitted molecules from SN and CN is given as:*

$$\min_{Q_s[j]}\quad \overline{P}_e[j+1]\tag{6}$$

$$\textit{s.t.}\quad \eta_d[j{+}1]-\lambda_{egc}^{10}[j{+}1]\leq 0,\eta_d[j{+}1]-\lambda_{egc}^{11}[j{+}1]\leq 0,\tag{7}$$

$$-\eta_d[j{+}1]+\lambda_{egc}^{00}[j{+}1]\leq 0,\eta_c[j]-\lambda_{sc}^{1}[j]\leq 0,\tag{8}$$

$$-\eta_d[j{+}1]+\lambda_{egc}^{01}[j{+}1]\leq 0,-\eta_c[j]+\lambda_{sc}^{0}[j]\leq 0.\tag{9}$$

Proof. The first derivative of $P_e[j+1|\mathbf{X}_1^{j-1}]$ after substituting $Q_c[j]{=}Q_t[j]{-}Q_s[j]$ in (5), is derived as,

$$\Delta(Q_s[j])\triangleq\frac{\partial A}{\partial Q_s[j]}+\frac{\partial B}{\partial Q_s[j]}+\frac{\partial C}{\partial Q_s[j]}+\frac{\partial D}{\partial Q_s[j]},$$

where

$$\frac{\partial A}{\partial Q_s[j]} = -\frac{\beta_1}{2\sqrt{\pi}}\Big\{(p_0^{sc}+\tau_1)\Lambda\left(\eta_c[j],\lambda_{sc}^1[j]\right)\left[1+\Xi\left(\eta_d[j+1],\lambda_{egc}^{10}[j+1]\right)\right] + (p_0^{sd}+\tau_2-\tau_3)\Lambda\left(\eta_d[j+1],\lambda_{egc}^{10}[j+1]\right)\left[1+\Xi\left(\eta_c[j],\lambda_{sc}^1[j]\right)\right]\Big\}, \quad (10)$$

$$\frac{\partial B}{\partial Q_s[j]} = \frac{\beta_1}{2\sqrt{\pi}}\Big\{(p_0^{sc}+\tau_1)\Lambda\left(\eta_c[j],\lambda_{sc}^1[j]\right)\left[1+\Xi\left(\eta_d[j+1],\lambda_{egc}^{11}\right)\right] - (p_0^{sd}+\tau_2-p_0^{cd}-\tau_3)\Lambda\left(\eta_d[j+1],\lambda_{egc}^{11}\right)\left[1-\Xi\left(\eta_c[j],\lambda_{sc}^1[j]\right)\right]\Big\}, \quad (11)$$

$$\frac{\partial C}{\partial Q_s[j]} = \frac{\beta_0}{2\sqrt{\pi}}\Big\{-\tau_1\Lambda\left(\eta_c[j],\lambda_{sc}^0[j]\right)\left[1-\Xi\left(\eta_d[j+1],\lambda_{egc}^{00}\right)\right] + (\tau_2-\tau_3)\Lambda\left(\eta_d[j+1],\lambda_{egc}^{00}[j]\right)\left[1+\Xi\left(\eta_c[j],\lambda_{sc}^0[j]\right)\right]\Big\}, \quad (12)$$

$$\frac{\partial D}{\partial Q_s[j]} = \frac{\beta_0}{2\sqrt{\pi}}\Big\{\tau_1\Lambda\left(\eta_c[j],\lambda_{sc}^0[j]\right)\left[1-\Xi\left(\eta_d[j+1],\lambda_{egc}^{01}\right)\right] + (\tau_2-p_0^{cd}-\tau_3)\Lambda\left(\eta_d[j+1],\lambda_{egc}^{01}\right)\left[1-\Xi\left(\eta_c[j],\lambda_{sc}^0[j]\right)\right]\Big\}, \quad (13)$$

here $\tau_1 \triangleq \sum_{i=1}^{j-1} x[j-i]p_i^{sc}, \tau_2 \triangleq \sum_{i=1}^{j-1} x[j-i]p_i^{sd}, \tau_3 \triangleq \sum_{i=2}^{j} \widehat{x}[j-i+2]p_{i-1}^{cd}$, and $\Lambda(\eta,\lambda) = \exp\left(-\frac{(\eta-0.5-\lambda)^2}{\sqrt{2\lambda}}\right)\left(\frac{\lambda+0.5+\eta}{(2\lambda)^{3/2}}\right)$. After differentiating (10)-(13), the constraints given in (7)-(8) can be obtained. □

REFERENCES

[1] H. B. Eriksson, P. Odling, T. Koski, and P. O. Borjesson, "A genie-aided detector with a probabilistic description of the side information," in *Proceedings of 1995 IEEE Int. Symp. on Info. The.* IEEE, 1995, p. 332.